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MATHEMATICAL ANALYSIS OF CYLINDRICAL BENDING OF PLATES CONSIDERING TRANSVERSE SHEAR DEFORMATIONS

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Abstract

The article examines the cylindrical bending of a plate along its surface for various boundary conditions, taking into account shear deformations. The relationships between bending- and shear-induced deflections and the effect of shear deformations on bending moments are established. Results are applicable to the analysis of three-layer panels and thin-walled aviation structures.

Keywo'rds: plastikna; deformatsiya; siljish; egilish; bukilish; momentlar; chegaraviy shartlar.

Introduction

In aeronautical engineering, the effect of transverse shear deformations must be taken into account when the transverse shear modulus of the material is small. This approach is applied to the analysis of three-layer panels — including acoustic sound-absorbing structures and thin-walled elements of aviation structures.

Problem Formulation.

The transverse bending of plates, accounting for shear deformations, is described by two functions: the transverse deflection $w(x,y)$ and the shear function $\varphi(x,y)$. These functions are related to each other and to the applied loads by a system of differential equations [1, 2]:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=-\frac{2}{3\cdot h}\cdot q$$

$$D_x\cdot\frac{\partial^4 w}{\partial x^4}+2\cdot D_0\cdot\frac{\partial^4 w}{\partial x^2\partial y^2}+D_y\cdot\frac{\partial^4 w}{\partial y^4}=q+\frac{4}{5}\cdot\left(\frac{D_x}{G_{XZ}}\cdot\frac{\partial^4\varphi}{\partial x^4}+D_0\cdot\left(\frac{1}{G_{XZ}}+\frac{1}{G_{YZ}}\right)\cdot\frac{\partial^4\varphi}{\partial x^2\partial y^2}+\frac{D_y}{G_{YZ}}\cdot D_x\cdot\frac{\partial^4\varphi}{\partial y^4}\right)\quad(1)$$

where D_x va D_y — silindrik stiffnesses; D_k — burilish stiffness; h — qalinlik; q — ko'ndalang distributed load; G_x va G_y — siljish moduli.

In deriving these equations, it is assumed that the shear stresses vary through the plate thickness [1] according to a quadratic parabolic law:

$$\tau_{XZ}=\frac{\partial\varphi}{\partial x}\left(1-4\cdot\frac{z^2}{h^2}\right);\tau_{YZ}=\frac{\partial\varphi}{\partial y}\left(1-4\cdot\frac{z^2}{h^2}\right)\quad(2)$$

Cylindrical bending of the plate is considered with deformations along the Y-axis neglected. In this case the displacements depend on a single coordinate, and the system of equations (1) admits an analytical solution.

Analytical Analysis.

The cylindrical bending of the plate in the XOZ plane is considered (Fig. 1). $L>>b$, for the case of cylindrical bending, the system of equations (1) takes the following form:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=-\frac{2}{3\cdot h}\cdot q$$

$$D_x\cdot\frac{\partial^4 w}{\partial x^4}=q+\frac{4}{5}\cdot\left(\frac{D_x}{G_{XZ}}\cdot\frac{\partial^4\varphi}{\partial x^4}\right)\quad(3)$$

Differentiating the first equation twice with respect to x and substituting the result into the second equation, we obtain:

$$D_x\cdot\frac{\partial^4 w}{\partial x^4}=q+\frac{6}{5}\cdot\left(\frac{D_x}{G_{XZ}\cdot h}\cdot\frac{\partial^2\varphi}{\partial x^2}\right)\quad(4)$$

The coefficient accounts for the quadratic distribution of shear stresses. For a uniformly distributed transverse load, the solution of system (3) has the form (5), where the constants are determined from the boundary conditions.

$$\varphi(x)=-\frac{3\cdot q}{2\cdot h}\cdot\frac{x^2}{2}+C_1\cdot x+C_2$$

$$w(x)=-\frac{q\cdot x^4}{24\cdot D_x}+\frac{C_3x^3}{6}+\frac{C_2x^2}{4}C_5\cdot x+C_6\quad(5)$$

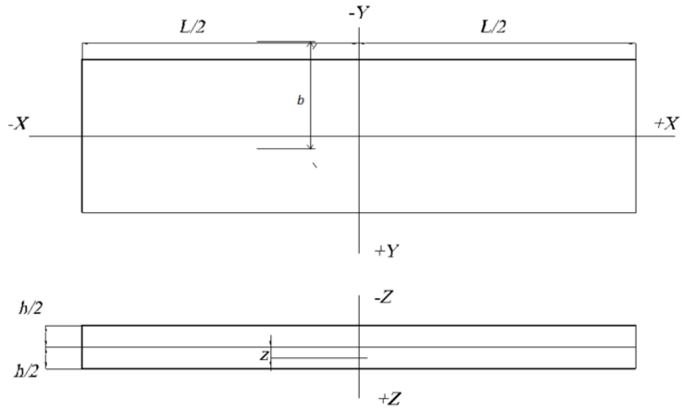

Fig. 1. Coordinate system and plate dimensions

$C_1, C_2, C_3, C_4, C_5, C$ The constants are determined from the boundary conditions at the plate edges.

To express the boundary conditions, the axial displacement of a plate layer at coordinate z is the sum of the rotation angles due to bending and shear [1], leading to expressions (7)–(11).

$$u_X(x,z) = -z\cdot\frac{\partial w}{\partial x} + \frac{1}{G_{XZ}}\cdot z\cdot\left(1-\frac{4}{3}\cdot\frac{z^2}{h^2}\right)\cdot\frac{\partial \varphi}{\partial x} \qquad (6)$$

Substituting equation (5) into expression (6), the axial displacement of a plate layer is found in form (7), and the relative axial displacement of the outer layers in form (8).

$$u(x,\frac{h}{2}) = \pm\frac{h\cdot\left(C_1-\frac{3\cdot q\cdot x}{2\cdot h}\right)}{3G_{XZ}} \mp \frac{h\cdot\left(C_5+\frac{C_3\cdot x^2}{2}+C_4\cdot x+\frac{q\cdot x^3}{6\cdot D_X}\right)}{2}$$

(7)

The plate's relative axial displacement of the outer layers (8), the bending moment (9) and the transverse shear force (10) are determined from the following expressions.

$$u(x,y) = \frac{u\left(x,\frac{h}{2}\right)-u\left(x,-\frac{h}{2}\right)}{2} = \frac{h\cdot\left(C_1-\frac{3\cdot q\cdot x}{2\cdot h}\right)}{3G_{XZ}} \mp \frac{h\cdot\left(C_5+\frac{C_3\cdot x^2}{2}+C_4\cdot x+\frac{q\cdot x^3}{6\cdot D_X}\right)}{2}$$

(8)

OY Bending moment about the axis:

$$M_Y = -D_X\cdot\left(\frac{\partial^2 w}{\partial x^2}\right)+\frac{4}{5}\cdot D_X\cdot\left(\frac{1}{G_{XZ}}\cdot\frac{\partial^2\varphi}{\partial x^2}\right) \qquad (9)$$

Expression for the transverse shear force:

$$Q_{XZ} = \frac{2 \cdot h}{3} \cdot \frac{\partial \varphi}{\partial x} \qquad (10)$$

Expression for curvature accounting for (9) and (10):

$$\frac{\partial^2 w}{\partial x^2} = -\frac{1}{D_X} \cdot M_Y + \frac{6}{5} \cdot \left(\frac{1}{G_{XZ}} \cdot \frac{\partial Q_{XZ}}{\partial x} \right) \qquad (11)$$

In expression (11), D_b is the bending stiffness and D_s is the shear stiffness. The following types of boundary conditions are considered in the article:

1) Simply supported edges: the transverse displacements and bending moments at the end sections are zero (Type 1 boundary conditions),

$$X = \frac{L}{2}; w\left(\pm\frac{L}{2}\right) = M_Y\left(\pm\frac{L}{2}\right) = 0$$

2) Clamped end sections: the transverse displacements and the total rotation angle of the cross-sections (sum of rotation angles due to shear and bending) are zero (Type 2 boundary conditions),

$$X = \frac{L}{2}; w\left(\pm\frac{L}{2}\right) = M_Y\left(\pm\frac{L}{2}\right) = 0; -\frac{\partial w}{\partial x} + \frac{1}{G_{XZ}} \cdot \frac{\partial \varphi}{\partial x}$$

3) Simply supported edges with axial displacement of the outer layers restrained (Type 3 boundary conditions),

$$X = \frac{L}{2}; w\left(\pm\frac{L}{2}\right) = M_Y\left(\pm\frac{L}{2}\right) = 0; h \cdot w\left(\pm\frac{L}{2}\right) - \frac{2 \cdot h}{3} \cdot \frac{\partial \varphi\left(\pm\frac{L}{2}\right)}{\partial x} = 0$$

4) Clamped end sections: the transverse displacement and the bending rotation angle are zero (Type 4 boundary conditions) and the rotation angle due to shear deformations is unconstrained,

$$X = \frac{L}{2}; w\left(\pm\frac{L}{2}\right) = M_Y\left(\pm\frac{L}{2}\right) = 0; -\frac{\partial w}{\partial x}\left(\pm\frac{L}{2}\right) = 0$$

Table 1 presents the transverse deflection equations due to bending and shear for all four types of boundary conditions. All expressions for the bending-induced deflection of the plate coincide with the corresponding beam-bending expressions.

The coefficient α is introduced to evaluate the ratio of the maximum shear-induced deflection to the maximum bending-induced deflection (Table 1).

$$\alpha = \frac{f_{maks.siljish}}{f_{maks.egilish}} \qquad (12)$$

Table 1 Equations for transverse deflection and bending moment in cylindrical bending of a plate

Type of boundary conditions	Transverse deflection due to bending	Transverse deflection due to shear	α	Bending moment
Sharnirli mustahkam-langan plate	$\frac{q}{24\cdot D_X}\cdot\left(\frac{5\cdot L^4}{16}-\frac{3\cdot L^2\cdot x^2}{2}+x^4\right)$	$\frac{3\cdot q}{5\cdot G_{XZ}\cdot h}\cdot\left(\frac{L^2\cdot x^2}{4}-x^2\right)$	$\frac{11{,}52\cdot D_X}{L^2\cdot G_{XZ}\cdot h}$	$\frac{q\cdot L^2}{8}-\frac{q\cdot x^2}{2}$
Clamped (total rotation angle = 0)	$\frac{q}{24\cdot D_X}\cdot\left(\frac{5\cdot L^4}{16}-\frac{3\cdot L^2\cdot x^2}{2}+x^4\right)$	$\frac{3\cdot q}{4\cdot G_{XZ}\cdot h}\cdot\left(\frac{L^2\cdot x^2}{4}-x^2\right)$	$\frac{72\cdot D_X}{L^2\cdot G_{XZ}\cdot h}$	$\frac{q\cdot L^2}{24}-\frac{q\cdot x^2}{2}+\frac{3\cdot q\cdot D_X}{10\cdot G_{XZ}\cdot h}$
Simply supported with axial displacement of outer layers restrained	$\frac{q}{24\cdot D_X}\cdot\left(\frac{5\cdot L^4}{16}-\frac{3\cdot L^2\cdot x^2}{2}+x^4\right)$	$\frac{q}{2\cdot G_{XZ}\cdot h}\cdot\left(\frac{L^2\cdot x^2}{4}-x^2\right)$	$\frac{48\cdot D_X}{L^2\cdot G_{XZ}\cdot h}$	$\frac{q\cdot L^2}{24}-\frac{q\cdot x^2}{2}+\frac{3\cdot q\cdot D_X}{5\cdot G_{XZ}\cdot h}$
Bending rotation angle = 0	$\frac{q}{24\cdot D_X}\cdot\left(\frac{5\cdot L^4}{16}-\frac{3\cdot L^2\cdot x^2}{2}+x^4\right)$	$\frac{3\cdot q}{5\cdot G_{XZ}\cdot h}\cdot\left(\frac{L^2\cdot x^2}{4}-x^2\right)$	$\frac{57{,}6\cdot D_X}{L^2\cdot G_{XZ}\cdot h}$	$\frac{q\cdot L^2}{24}-\frac{q\cdot x^2}{2}$

Table 1 also presents the bending moment expressions for all four types of boundary conditions. For Type 1 and Type 4 boundary conditions the bending moment is independent of the shear modulus; for Type 2 and Type 3 conditions an additional bending moment component arises due to shear deformations.

Table 2 presents the equations for the relative axial displacement of the plate's outer layers, which consists of two components: parts due to bending and due to shear.

Table 2 Equations for the relative axial displacement of the plate's outer layers

Type of boundary conditions	Relative axial displacement of the plate's outer layers
Sharnirli mustahkamlangan plate	$u(x)=\frac{q\cdot x\cdot L^2\cdot h}{16\cdot D_X}-\frac{q\cdot x^3\cdot h}{12\cdot D_X}+\frac{q\cdot x}{10\cdot G_{XZ}}$
Clamped (total rotation angle = 0)	$u(x)=\frac{q\cdot x\cdot L^2\cdot h}{16\cdot D_X}-\frac{q\cdot x^3\cdot h}{12\cdot D_X}+\frac{q\cdot x}{10\cdot G_{XZ}}$
Simply supported with axial displacement of outer layers restrained	$u(x)=\frac{q\cdot x^3\cdot h}{12\cdot D_X}+\frac{q\cdot x\cdot L^2\cdot h}{48\cdot D_X}$
Bending rotation angle = 0	$u(x)=\frac{q\cdot x\cdot L^2\cdot h}{48\cdot D_X}-\frac{q\cdot x^3\cdot h}{12\cdot D_X}+\frac{9\cdot q\cdot x}{40\cdot G_{XZ}}$

For plates with thicknesses of 1, 10, 20 mm and lengths of 100, 200, 500 mm, $E = 7\times10^4$ kgf/mm², while G was varied from 1 kgf/mm² (lightweight filler) to 2.6×10^4 kgf/mm² (metal).

The dependence of the ratio of bending moments calculated with and without shear deformations on the shear modulus was studied for Type 2 and Type 3 boundary conditions.

The analysis shows that for thin and long plates the influence of the shear modulus is negligible (less than 1% for h = 1 mm, l = 500 mm); for thick and short plates this influence increases significantly.

Minimum values of the shear modulus have been calculated for cases where the shear-induced bending moment constitutes 0.5% of the bending-induced moment; these values define the range of applicability of the theory.

When the ratio of shear-induced to bending-induced bending moments exceeds 0.5%, this approach becomes inapplicable, as the sum of moments at the end sections differs significantly from zero.

The analysis shows that for the simply supported plate (Type 1) the influence of the shear modulus is smallest; for Type 2–4 boundary conditions, variations in the shear modulus have nearly the same effect on the deflection ratio.

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Conclusion

1) For Type 2 and Type 3 boundary conditions, when shear rotation is constrained, additional bending moments arise whose magnitudes depend on the ratio D_s/D_b. In this case the conditions of static equilibrium are violated because the sum of the additional moments differs from zero.
2) For plates of various dimensions, the minimum values of the shear modulus required for static equilibrium to be satisfied to within 0.5% accuracy under Type 2 and Type 3 boundary conditions have been calculated — this defines the range of applicability of the presented theory.
3) The greatest influence of the shear modulus on bending is observed for Type 2–4 boundary conditions, for which the magnitude of the shear modulus has nearly equal effect on the ratio of bending- and shear-induced deflections.

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